PROBABILISTIC METHODS FOR THE EEG INVERSE PROBLEM

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Abstract: **In the common approach to EEG dipole modeling, one typically searches for the solution that fits best the observed data for a given number of sources. Instead of searching for this single 'best' solution, we propose a method that attempts for each combination of a few dipoles to infer whether these are plausible sources given the observed EEG potentials. We use classical statistical tests, thus enabling us to list the sets of sources by order of significance and build probabilistic maps. We present the method followed by a simulation on a 2-dipole case.**

INTRODUCTION

The spatial distribution of EEG potentials can be used to infer the location of neuronal sources of events such as epileptic spikes. However, the inverse problem is ill posed, which means that an infinite number of different source configurations can produce the exact same potentials on the scalp. This requires the use of spatial constraints on the sources. One approach is to restrict the source model to just one or several current dipoles. In the noiseless situation, with a properly designed captor array, the solution would then be unique [1]. Unfortunately, in the real world situation the array is not ideal (sparse sampling and the captors only enclose the upper half of the brain) and the data are contaminated by noise. This results in an ambiguous problem.

Traditionally, for a given potential distribution, one would select the maximum likelihood (ML) solution. For example, with the assumption of a white Gaussian additive noise, the ML solution is that which minimizes the residual sum of squares [2].

Recently, Schmidt et al. [3] have suggested assessing the range of likely solutions. They used a Bayesian framework to generate a probability for each solution. They used Markov Chain Monte Carlo (MCMC) simulations to build statistical maps of the probability of a given point to contain a source.

We propose here to consider solutions that consist of only one to three dipolar sources, which is probably a reasonable assumption in the case of epileptic spikes. We use a coarse grid (1 cm spacing), which is still a useful resolution in presurgical evaluation of epileptic patients and avoids MCMC computations by enabling a permutation approach. We present a method that evaluates the probability of a given combination of dipoles to be consistent with the measured potentials. This method tests the hypothesis that the model fits well the data but also that all dipoles in the combination are useful. This is a more strict rating of the combinations of sources than [3].

METHODS

Probabilistic model

We assume that the observed scalp potential distribution is the sum of the contributions of each source,

$$Y = A(n,\theta)B + E \tag{1}$$

with: Y observed potentials (#electrodes x 1); n number of sources; θ source coordinates; A(n,θ) matrix (#electrodes x 3n) containing 3 orthogonal unit dipoles per location; B source strength (3n x 1); E Gaussian noise with known covariance Σ (here the noise components are assumed to be independent and so Σ is diagonal). For a given (n, θ), we take the weighted least squares estimate of B,

$$\hat{B} = (A^T\Sigma^{-1}A)^{-1}A^T\Sigma^{-1}Y \tag{2}$$

which corresponds to the minimization of the weighted residual sum of square,

$$SSQ = (Y - AB)^T\,\Sigma^{-1}(Y - AB) \tag{3}$$

Test 1. With the assumption of known Gaussian noise, the likelihood of the data for a given set of parameters (i.e. we test that the model fits well the data) is:

$$L = \frac{1}{\sqrt{(2\pi)^d\,|\Sigma|}}\,e^{-\frac{1}{2}(Y-AB)^T\,\Sigma^{-1}(Y-AB)} \tag{4}$$

with d= #electrodes, |Σ| determinant of Σ.

Test 2. We test the significance of increasing the model order using an F-test. The SSQ given by (3) is tested against the best (minimum) SSQ at order (n-1) (i.e. we test that adding a dipole significantly improves the model). The test is:

$$F = \frac{(SSQ_{\mathbf{min}}^{n-1} - SSQ)/3}{SSQ/(d-3n)} \tag{5}$$

following an F distribution with 3 and (d-3(n-1)) d.f.

Test 3. We combine test 1 with examining the contribution from each dipole to the fit (i.e. we test that the model fits the data and that each individual source contributes significantly to the combination). This is done by replacing SSQ_{min}^{n-1} in (5) by the SSQ obtained by neglecting the contribution of this source.

One can then build probabilistic maps by integrating solutions derived from these tests across the dipole combinations.

Simulation

We simulated two radial dipoles at location (x,y,z) = (-5, 5, 55) and (45, 5 ,45) within a three-sphere models with scalp, skull and brain radii of 92, 85 and 80mm respectively (x axis from back to front, y axis from right to left, z axis pointing up) (Fig. 1). We used 71 channels (10/10 system).

We added one realization of white noise scaled so as to produce a SNR of 20 (linear scale). We tried all combinations of one, two and three dipoles on a grid (one sagittal plane, 1cm spacing, fig. 1). For each combination, we evaluated formula (4) and (5) and constructed two sets of maps,

$$MAP_1(x_0, z_0) = \frac{1}{C^2_{npts}} \sum_{\theta} L(\theta) \quad I\{(x_0, z_0) \in \theta\}$$

$$MAP_2(x_0, z_0) = \frac{1}{C^3_{npts}} \sum_{\theta} I\{(x_0, z_0) \in \theta \;\&\; F > t_f(n)\}$$

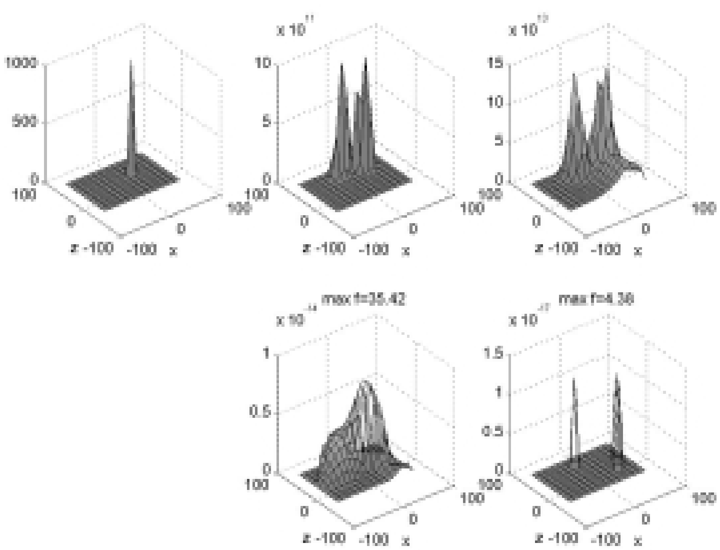
with I{B}=1 if B is true, I{B}=0 if B is false; t_f threshold for the F-test at p=0.01 (no correction for multiple comparison); npts number of grid points (here, npts=140).

Here, $t_f(n=2) = F^{-1}(p=0.01, df1=3, df2=71-6) = 4.09$

$t_f(n=3) = F^{-1}(p=0.01, df1=3, df2=71-9) = 4.11$

RESULTS

Figure 2 presents the maps obtained on the simulated data for n=1,2 and 3. Map 1 has one peak for n=1, two peaks at the correct source locations for n=2 and n=3. Map 2 has only 3 non-zero points for n=3 and the maximum F test value from (5) is much lower for n=3, showing that a model with 3 sources does not improve significantly on the 2 sources model. The integration of test 3 produced very similar results to MAP 1 (not shown).



n=1 n=2 n=3

Fig. 2. Probabilistic maps obtained on the simulated dipoles (SNR of 20). Upper row: map 1. Lower row: map 2. Columns: number of sources in the scanning.

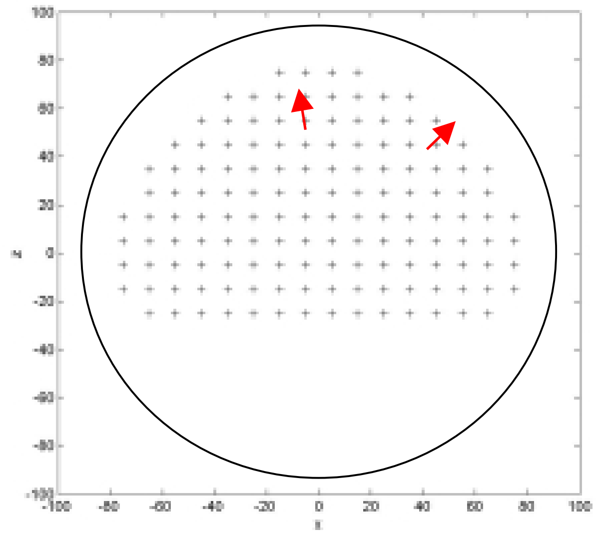


Fig. 1: Two dipoles inside the scalp sphere (x axis from back to front of head). The scanning grid is shown with crosses.

DISCUSSION

We have presented an approach that uses statistical methods to assess possible sources to EEG potentials. The number of parameters has been minimized. This is somewhat contrary to the current tendency of trying to determine the source locations with a very high precision, a difficult task in a noisy situation.

All three tests will provide information on the ambiguity of the problem, as the generated maps will become flatter with less information revealed when the problem is very ill-defined. Test 2 offers additional information on the model order (number of dipoles) that is supported by the data. Test 3 could be more selective than test 1 alone in some situations.

Our simulation suggests that the use of model selection techniques is a relevant approach in the EEG inverse problem and can add information to the classical measure of likelihood.

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